APPLICATION OF QUEUING THEORY IN CADET MESS ADMINISTRATION: A CASE STUDY OF NIGERIAN DEFENCE ACADEMY, KADUNA NIGERIA

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Abstract
The application of Queuing models as a Technique of Queue solution in Mess System was carried out in Nigerian Defence Academy, Kaduna. Specifically, this study attempts to look at the problem of long queues in cadet mess, why mess managers find it difficult to eliminate queues and the effect of queuing model as a technique of queue solution in Mess System. The variables measured include arrival rate (λ) and service rate (μ). They were analyzed for simultaneous efficiency in cadet satisfaction and cost minimization through the use of a multichannel queuing model, which were compared for a number of queue performances. It was discovered that, using a six-server (six-channel) system was better than a 3-server, 4-server or 5-server systems in terms of the performance criteria used and the study inter-alia recommended that, the management should maintain a six-server model to increase cadet satisfaction.

Keywords: *Cadet Mess, Multi-Channel Queuing Model, Officer Cadet, Queue length, waiting time.*

*** A retired Commodore of the Nigerian Navy*

INTRODUCTION

"The Nigerian Defence Academy (NDA) is the premier officer cadets military university in Nigeria, established to provide each officer cadet with knowledge, skill and values necessary to meet the requirements of the military officer through military training, academic excellence and character development" New Valiant (2011). An officer cadet is a student in the armed forces or the police (According to Cambridge Advanced Learner's Dictionary [2008]). Mess is a room or building in which members of the armed forces have their meals or spend their free time, Cambridge Advanced Learner's Dictionary (2008). The order through which the officer cadets of the NDA observe their meal in the mess attracts waiting line which is cumbersome, time consuming and tedious. Thus, the need to consider the problem of waiting line (queue) in the cadet mess, the average number of cadets arrival per unit time (λ) or inter arrival between two cadets (1/λ), the average number of cadets being served per unit time (μ) or service time between two cadets (1/μ), service channels, length of queue, queue discipline, maximum number allowed in the system and size of the calling source.

Waiting in line (queue) is certain in a lot of service areas. Sundarapandian, (2009) states that Queuing theory started with research by Erlang when he created models to describe the Copenhagen telephone exchange. The idea of queuing theory can be traced back to the classical work of Erlang in 1900s, however the work of Kendal in 1951 formed the basis for analytical calculations and the naming convention in queues being used today, Dombacher (2010). Sundarapandian, (2009) Queuing theory is the mathematical study of waiting lines, or queues. Sharma (2015), defined queue as a general phenomenon in

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everyday life. Queues are formed when customers (human or not) demanding service have to wait because their number exceeds the number of servers available at a given time or the facility doesn't work efficiently or takes more than the time prescribed to service a customer. Queuing psychology recognizes that the cadet's cost of waiting is not just about the time cadets spend waiting in line, but includes what cadets think about the waiting. Agyei *et.al.,* (2015), Some customers wait when the total number of customers requiring service exceeds the number of service facilities, some service facilities stand idle when the total number of service facilities exceeds the number of customers requiring service. Crowley *et al.,* (1995), present a queuing analysis performed during the initial design of a production facility for electromechanical devices.

Nosek and Wilson (2001), stated that queuing theory utilizes mathematical models and performance measures to assess and hopefully improve the flow of customers through a queuing system. In queuing theory a model is constructed so that queue lengths and waiting times can be predicted, Sundarapandian, (2009). Taha (2003), defines queue as simply a waiting line, while Hiray (2008), puts it in similar way as a waiting line by two important elements: the population source of customer from which they can draw and the service system. The population of customer could be finite or infinite Johye *et.al.,* (2010), many restaurant chains and fast food industry outlets use waiting time standards as an explicitly advertised competitive edge.

Statement of Problem

Based on their scheduled programs, all officer cadets are expected to go for their meals at the same time which could result in population upsurge and waiting line in the mess hall. As a consequence of queue length, the total time officer cadets spend in queue plus the service time is also affected. Hence, may affect their next program. In a bid to meet their programs it could cause problems like balking, reneging, collusion or jockeying.

All cadets need to feed well to able to meet the entire academic and physical training. The problem is employing a model that ensures minimum time in the mess and cadets maintain prescribed queuing discipline.

Direction of current effort

The current effort attempts to track down the effect of waiting time, if there are alterations in the facilities available. It then proposes a suitable queue model that minimizes queue length in the mess system which will improve the cadet mess service facility. It is envisaged that this study will contribute immensely in the analysis of reducing waiting time in the mess hall and help to attract more cadets to avail themselves of the mess services.

II MATERIAL AND METHODS

The queuing system consists essentially of three major components: (1) the source population and the way cadets arrive at the system, (2) the servicing system, and (3) the condition of the cadets exiting the system. The system consists of more servers, an arrival pattern of cadets, service pattern, queue discipline, the order in which services are provided and cadet behavior.

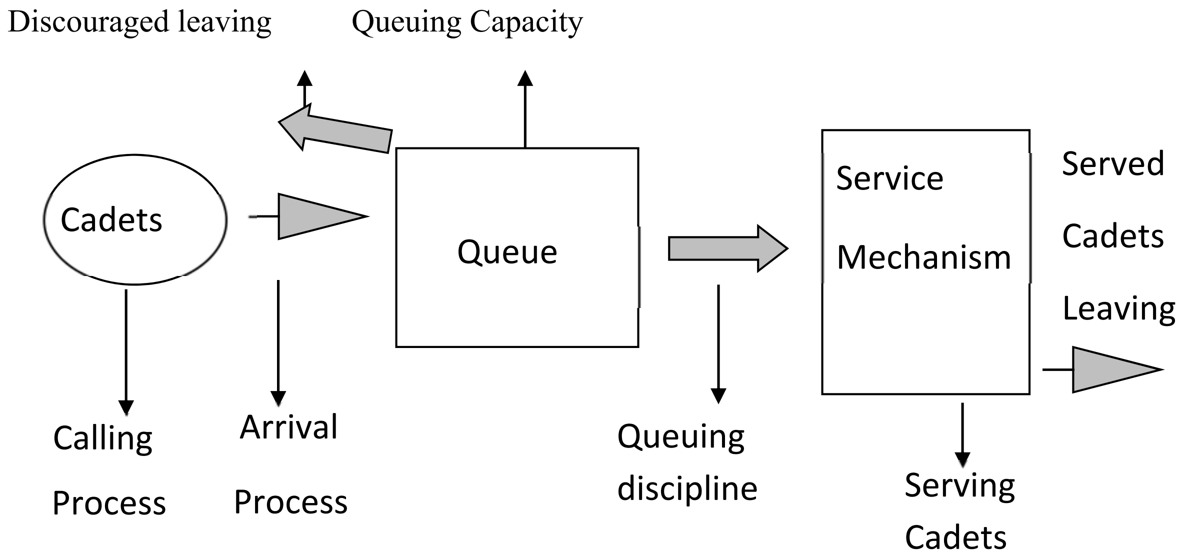

Figure 1: General Overview of Queuing System

Multiple Queue and Multiple Servers

This can also be called Single Stag Queue in parallel as described in Figure 2. It is similar to that of Single Queue – Server Queue, only that there are many servers performing the same task with each having a queue to be served. This type of queue is practiced in the Nigerian Defence Academy, Cadet Mess, Kaduna.

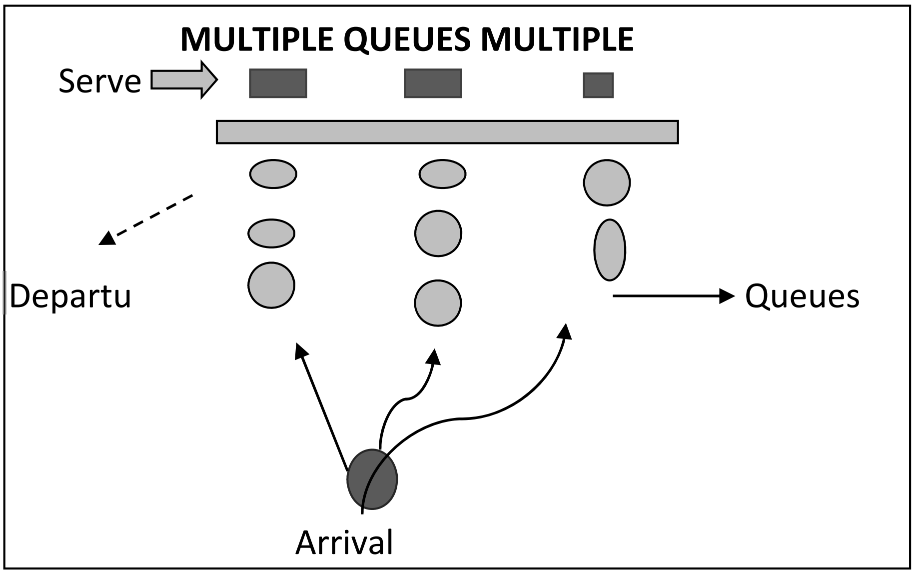

Figure 2: Extant Queuing Structure of NDA Cadet Mess

Method

a. LITTLE'S LAW

According to Little (1961), The long-term average number of customers in a stable system L, is equal to the long-term average arrival rate, , multiplied by the long-term average time a customer spends in the system, W; i.e. .

b. NOTATION FOR QUEUES

Since all queues are characterized by arrival, service and queue and its discipline, the queue system is usually described in shorten form by using the general notation: {A/B/s}:{d/e/f}

Where,

A= probability distribution of the arrivals, B= probability distribution of the departures, S = number of servers (channels), d = the capacity of the queue(s), e = the size of the calling population, f = queue ranking rule (ordering of the queue).

c. M/M/s MODEL

The description of a M/M/s queue is similar to that of the classic M/M/1 queue with the exception that there are s servers. When s=1, all the result for the M/M/1 queue can be obtained. The number of cadets in the system at time t, x(t), in the M/M/s queue can be modeled as a continuous times Markov chain.

The condition for stability is $p = \frac{\lambda}{s\mu} < 1$ where λ is mean arrival rate, μ is mean service rate, s is number of servers and p is called the service utilization factor or the proportion of time on average that each server is busy. The total service rate must be greater than the arrival rate, that is $s\mu > \lambda$, and if $s\mu \leq \lambda$ the queue would eventually grow infinitely large.

The probability that at any given time there are no cadets waiting or being served at steady state

$$P_0 = \left(\sum_{j=0}^{s-1} \frac{(sp)^j}{j!} + \frac{(sp)^s}{s!\,(1-p)} \right)^{-1}$$

Where:

S = number of servers

p = service utilization factor

j = range of server (s) for j = 0, 1, 2, ... , s-1

The average number of cadets waiting in queue to be served L_q.

$$L_q = P_0 \frac{s^s p^{s+1}}{S!\,(1-p)^2}$$

The average number of cadets in service L_s,

$$L_s = \sum_{j-1}^{s-1} jP_j + \sum_{j=s}^{\infty} sP_j = sp$$

The average number of cadets in the system becomes

$$L = L_q + L_s = L_q + sp = L_q + \frac{\lambda}{\mu}$$

The average time cadets spend in waiting in queue before service starts $W_q$is

$$W_q = \frac{L_a}{\lambda}$$

The average time cadets spend in the system, waiting plus being served W is

$$W = \frac{L}{\lambda} \frac{L_a + \frac{\lambda}{\mu}}{\lambda} = \frac{L_a}{\lambda} + \frac{1}{\lambda} = W_q + \frac{1}{\mu}$$

The average time cadets are served

$$W_s = W - W_q = \left(W_q + \frac{1}{\mu}\right) - W_q = \frac{1}{\mu}$$

III DATA ANALYSIS AND DISCUSSION OF RESULTS

Microsoft-office plus. 2013(excel solver) was used for the computation, Analysis and summary of results of the data are presented and discussed

The queuing model used in the analysis is *M/M/s* which involves a single-line with multiple servers in the system.

The following assumptions are made:

1. The cadets face balking, reneging, or jockeying and come from a population that can be considered as infinite.
2. Cadets arrivals are described by a Poisson distribution with a mean arrival rate of (lambda). This means that the time between successive cadet arrivals follows an exponential distribution with an average of $1/\lambda$.
3. The cadets service rate is described by a Poisson distribution with a mean service rate of μ (mμ). This means that the service time for one cadet follows an exponential distribution with an average of 1/μ.
4. The waiting line priority rule used is first-come, first-served. Using these assumptions, we can calculate the operating characteristics of a waiting line system.

SOURCE OF DATA

The data was collected from the Nigerian Defence Academy, Kaduna on different days and at times during breakfast, lunch and dinner which involve arrival and service time of cadets. The data was collected within some randomly selected meals, so as to check whether cadets face the same situation at any time they enter the mess hall for their meal. The collection was based on the number of cadet‘s arrival time and service time. The data was collected with an average of one hour during the days of 14th, 16th, 25th and 29th all in the month of January, 17th, 21st, 28th were also for the month of February and 2nd March all in year 2015.

Table 1. Shows Primary Data Summary For the Randomly Selected Hours and Days

Date	Time range	Arrival Rate	No. of Servers	Service Rate	Remark
14th Jan	0645hr –0745hr	520	3	187	
16th Jan	1500hr – 1600hr	980	3	339	
25th Jan	0645 hr – 0745hr	650	3	230	
29th Jan	1500 hr – 1600hr	720	3	253	
17th Feb	1500 hr – 1600hr	752	3	265	
21st Feb	1500 hr – 1600hr	780	3	273	
28th Feb	0645 hr – 0745hr	680	3	244	
2nd Mar	1900 hr –2000hr	580	3	206	

Results for sample computation are shown below for the data of 14th January, The results for other dates follow similar calculations using excel solver.

1. Utilization factor for 14th January is given by:
ρ = $p = \frac{\lambda}{s\mu} = \frac{520}{3 \times 187}$ = 0.9269

2. The probability that at any given time the system will be idle (there are no cadets waiting).

$P_0 = \left(\sum_{j=0}^{s-1} \frac{(sp)^j}{j!} + \frac{(sp)^s}{s!(1-p)}\right)^{-1} = \left(\sum_{j=0}^{3-1} \frac{(3 \times 0.9269)^j}{j!} + \frac{(3 \times 0.9269)^6}{3!(1-0.9269)}\right)^{-1}$

$= \left(\frac{(3 \times 0.9269)^0}{0!} + \frac{(3 \times 0.9269)^1}{1!} + \frac{(3 \times 0.9269)^2}{2!} + \frac{(3 \times 0.9269)^3}{3!(1-0.9269)}\right)^{-1}$

$$P_0 = (1 + 2.7807 + 3.8661 + 49.0223)^{-1}$$

P_0 =0.017642

3. Probability of an average number of cadets waiting in queue to be served L_q

$$L_q = P_0 \frac{S^s p^{s+1}}{S!\,(1-p)^2}$$

$$= 0.017642 \times \frac{3^3 (0.9269)^{3+1}}{3!\,(1-0.9269)^2}$$

L_q = 10.9719

4. The average number of cadets in the servers Ls

$Ls = \frac{\lambda}{\mu} = \frac{520}{187}$ = 2.7807

5. The average number of cadets in the system L

$$L = L_q + Ls$$

$$= 10.9719 + 2.7807$$

$$L = 13.7526$$

6. The average time a cadets spend in waiting in queue before service starts Wq is

$$W_q = \frac{L_q}{\lambda}$$

$= \frac{2.921348}{520}$ = 0.0211

7. The average time cadets are served Ws is

$W\text{s} = \frac{1}{\mu}$

$= \frac{1}{187}$ = 0.005348

8. The average time a cadets spends in the system, waiting plus served

$$W = W_q + \frac{1}{\mu} \quad but\ W_q = \frac{L_q}{\lambda}$$

$W = 0.0211 + 0.005348$ =0.02644

Table 2. Presentation on data analysis

Date	Jan_14	Jan_16	Jan_25	Jan_29	Feb_17	Feb_21	Feb_28	Mar_2	Remark
Arrival Rate	520	980	650	720	752	780	680	580	
Service Rate/Channel	187	339	230	253	265	273	244	206	
Number of Servers	3	3	3	3	3	3	3	3	
Type	M/M/3	M/M/3	M/M/3	M/M/3	M/M/3	M/M/3	M/M/3	M/M/3	
Mean Number at Station (L)	13.753	27.577	17.329	19.544	18.569	21.085	14.148	16.340	
Mean Time at Station (W)	0.0264	0.0281	0.0267	0.0271	0.0247	0.0270	0.0208	0.0282	
Mean Number in Queue (Lq)	10.972	24.687	14.502	16.698	15.731	18.227	11.361	13.524	
Mean Time in Queue (Wq)	0.0211	0.0252	0.0223	0.0232	0.0209	0.0234	0.0167	0.0233	
Mean Number in Service (Ls)	2.7807	2.8908	2.8261	2.8459	2.8377	2.8571	2.7869	2.8155	
Mean Time in Service (Ws)	0.0054	0.0030	0.0045	0.0040	0.0038	0.0037	0.0041	0.0049	
Efficiency (ρ)	0.9269	0.9636	0.9420	0.9486	0.9459	0.9524	0.9290	0.9385	
Probability All Servers Idle (ρ_o)	0.0176	0.0084	0.0138	0.0121	0.0128	0.0112	0.0171	0.0146	
Prob. All Servers Busy	0.8651	0.9320	0.8925	0.9045	0.8995	0.9114	0.8688	0.8861	

RESULTS FOR AVERAGE DATA

The total number of cadets observed was 5662, out of that 1997 was served and it a total of 8 hours with 3 servers. Table 3 below shows the intermediate calculations and performances measures.

Table 3. Average Data Calculation Result		Remark
INPUTS		
Total Time Involved (t)	8 hours	
Number of Cadets Arrived	5662	
Number of Cadets Served	1997	
Number of Servers	3	
Model Type	m/m/3	
PERFORMANCE MEASURES		
Rho(average server utilization),ρ	0.9434	
Probability of System empty, ρ_o	0.0135	
Average Cadets in the system, L	18.5431	
Average Cadets waiting in a queue, Lq	15.7128	
Average Cadet's wait in the service, Ls	2.830	
Average time in the system, W	0.0261	
Average time in the queue, Wq	0.0220	
Average time a customer is served, Ws	0.0042	

PROJECTIONS USING 25th JANUARY

Now let us consider one of the days in which the cadet mess recorded capacity utilization closer to total average server utilization. Studying the performance analysis assuming there is one-server to eight-servers. Compare their results to make policy recommendation. Use an average arrival rate, (λ) and service rate, (μ) of the 25th January for the analysis. The table 4 presents the results for considering one to eight servers at a given time using the inputs obtained on the 25th January year 2015.

Table 4. Shows Results of Types Models From One to Eight Servers at a Given Point.

Arrival Rate	650	650	650	650	650	650	650	650
Service Rate/Channel	230	230	230	230	230	230	230	230
Number of Servers	1	2	3	4	5	6	7	8
Type	M/M/1	M/M2	M/M/3	M/M/4	M/M/5	M/M/6	M/M/7	M/M/8
Mean No. at Station(L)	-1.5476	2.1627	17.3286	3.8825	3.0799	2.8958	2.8453	2.8311
Mean Time at Station(W)	-0.0024	0.0033	0.0267	0.0060	0.0047	0.0045	0.0044	0.0044
Mean No. in Queue(Lq)	-4.3737	-0.6632	14.5025	1.0564	0.2539	0.0697	0.0192	0.0050
Mean Time in Queue(Wq)	-0.0067	-0.0010	0.0223	0.0016	0.0004	0.0001	2.95E-05	7.76E-06
Mean No. in Service(Ls)	2.8261	2.8261	2.8261	2.8261	2.8261	2.8261	2.8261	2.8261
Mean Time in Service(Ws)	0.0043	0.0043	0.0043	0.0044	0.0043	0.0043	0.0043	0.0043
Efficiency(ρ)	2.8261	1.4130	0.9420	0.7065	0.5652	0.4710	0.4037	0.3533
Prob. All Servers Idle(ρ_o)	-1.8261	-0.1712	0.0138	0.0485	0.0565	0.0586	0.0591	0.0592

The average number of cadets waiting in the system and time they are served remain constant from one-server to eight-servers. The inappropriateness of a single server model for solving cadets – waiting time problems become apparent as it shows negative figures for all performance criteria accept (ρ), (Ls) and (Ws). However, multi – server models were compared and it is seen that;

- Using a six – server system with (ρ) is 0.4710 that is 47% of busy server which optimize both the waiting time and cost, that is, to strike a balance between waiting time and cost of employing more servers. Using a six – server system is better than a three – server system in all complicating result. For instance, assuming during that morning, there were three servers serving the cadets, there would have been 17.3286 cadets in queue system instead of 2.8958 cadets and the time spend in system is 0.0267 instead of 0.0045 hours respectively.
- A six – server system has a high probability of being idle 0.0586 than five – server, four – server and three server system.

THE PROPOSED MODEL

To formulate a suitable model with characteristics that will enable a solution to this problem of long wait be achieved, a modification of the original model in the area of number of channels was made thus applying the principle of (m/m/s) the multi-channel system. On analysis, this model proved workable as it produced the desired result of reducing queuing time. It is therefore presented here as the proposed model. *M/M/6/FCFS/∞/∞* .

This is a multi-channel queuing model with 6 channels, arrival and service times are both Poisson. The queue discipline is first come first serve. It has one queue from which cadet are allocated to the channels.

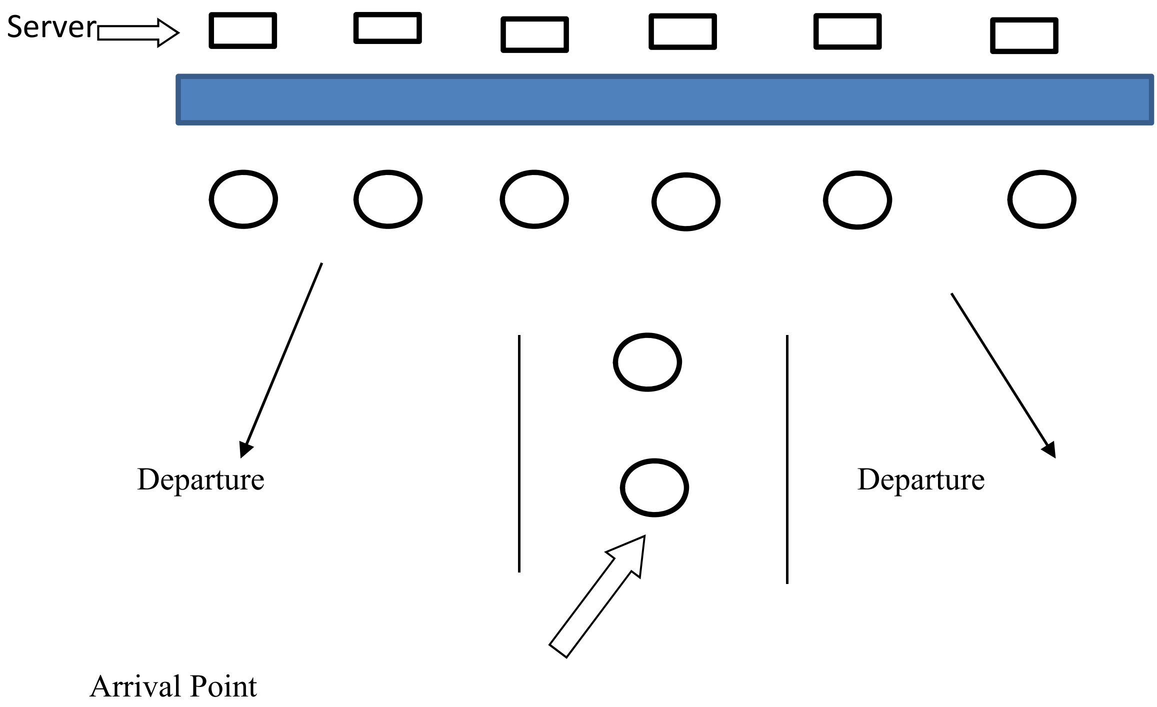

Figure: 3

DISCUSSION

The results show that the server would be busy 94.34% of the time and idle 1.35% of the time. Also, the average number of cadets in the queue is 16 and the average number of cadets in the system is 19. More so, the average time a cadet spends in the queue is 0.0220 hours and average a cadet spends in the system is 0.0261 hours.

It is determined using six-servers that cadets spent little time at the officer cadet mess system of Nigerian Defence Academy. From the result obtained, a cadet spent an average of 0.0261 hours that is 1.6 minute in the system. During their meal, they spend an average of 0.0246, 0.0267 and 0.0282hours for breakfast, lunch and dinner respectively. 2nd march recorded the highest waiting time spent in mess system with 0.0282hour and it was followed by 16th January with 0.0281 cadets as compared to 17th February and 28th February had the least waiting time in the system with 0.0270 and 0.0208 hours respectively as shown in table 2.

It is also observed that cadets waiting line (queuing length) is much at cadet mess of Nigeria Defence Academy if still using three-server system. The average number of cadets in the system from Table3 is 19 cadets will be in the system. For 16th January recorded the highest number of cadets the mess which is 28 cadets and it was followed by 21st of February that is 21 cadets.

CONCLUSION AND RECOMMENDATION

CONCLUSION

This study minimizes the amount of waiting time a cadet is likely to experience and thus reduce population upsurge in the mess system. The study uncovered the applicability and extent of usage of queuing models in achieving cadet satisfaction as well as permitting us to make better decisions relating to servers, length of queue and potential waiting times for cadets.

It was determined using six-servers from table 4.9; a cadet will spend 0.0045hour (16 sec.) waiting time in mess system. From table4.4; the result obtained, a cadet spents an average of 0.0261 hours that is (1.6 mins.) in the system.

It was determined using six-servers from table 4.9; they will be 3 cadets in queue system. It was also observed that cadets waiting line (queuing length) is much at cadet mess of Nigeria Defence Academy if still using three-server system which was 19 on the average from table 4.4.

During their meal, they spend an average of 0.0246, 0.0267 and 0.0282hours for breakfast, lunch and dinner respectively. They have an average queue length of 16, 22 and 16 for breakfast, lunch and dinner respectively.

RECOMMENDATIONS

Based on the summary and conclusion of this study, the following recommendations were made for efficient and quality service to officer cadets of the Nigerian Defence Academy, Kaduna.

1. The management should adopt a six-server model to reduce waiting time at the mess hall especially during their lunch which showed the highest population in order to increase cadet satisfaction.
2. An automatic replacement mechanism for servers that leave should be adopted.

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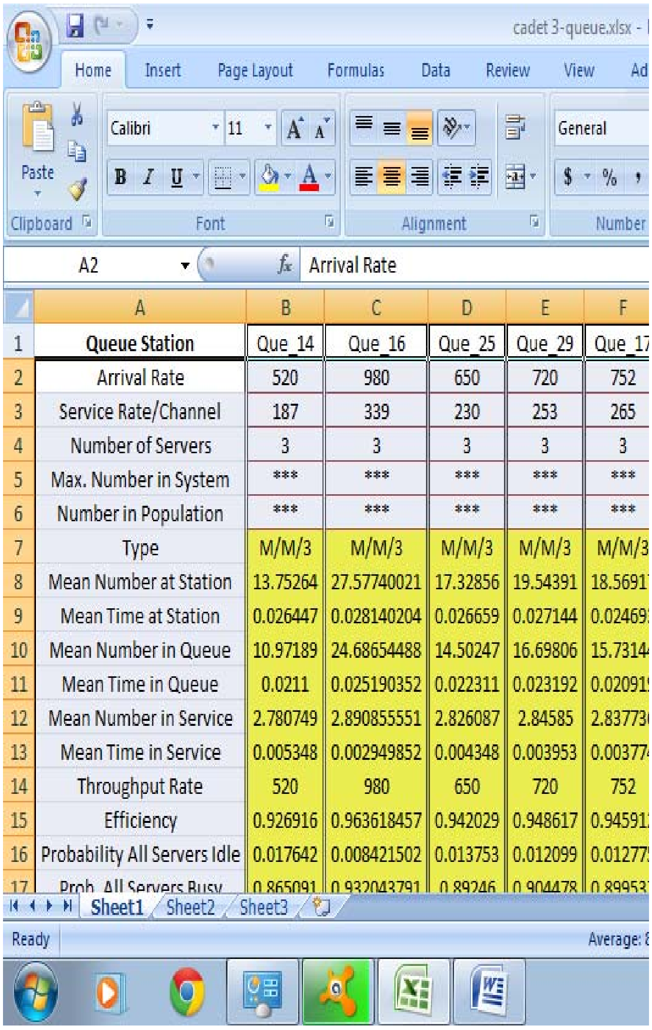
APPENDICES

A.2.0 SAMPLE OF MICROSOFT OFFICE EXCEL SPREADSHEET RESULT



	A	B	C	D	E	F
1	Queue Station	Que_14	Que_16	Que_25	Que_29	Que_17
2	Arrival Rate	520	980	650	720	752
3	Service Rate/Channel	187	339	230	253	265
4	Number of Servers	3	3	3	3	3
5	Max. Number in System	***	***	***	***	***
6	Number in Population	***	***	***	***	***
7	Type	M/M/3	M/M/3	M/M/3	M/M/3	M/M/3
8	Mean Number at Station	13.75264	27.57740021	17.32856	19.54391	18.5691
9	Mean Time at Station	0.026447	0.028140204	0.026659	0.027144	0.02469
10	Mean Number in Queue	10.97189	24.68654488	14.50247	16.69806	15.7314
11	Mean Time in Queue	0.0211	0.025190352	0.022311	0.023192	0.02091
12	Mean Number in Service	2.780749	2.890855551	2.826087	2.84585	2.83773
13	Mean Time in Service	0.005348	0.002949852	0.004348	0.003953	0.00377
14	Throughput Rate	520	980	650	720	752
15	Efficiency	0.926916	0.963618457	0.942029	0.948617	0.94591
16	Probability All Servers Idle	0.017642	0.008421502	0.013753	0.012099	0.01277
17	Prob. All Servers Busy	0.865091	0.932043791	0.89246	0.904478	0.89953